Assignment 8

Deadline: April 11, 2018.

Hand in: 1, 2 and 4

1. The Fourier transform maps $\mathcal{S}(\mathbb{R})$ to $\mathcal{S}(\mathbb{R})$.

2. Show that the Fourier transform of $e^{-a|x|}$ is equal to $2a/(\xi^2 + a^2)$.

3. Show that the Fourier transform of $\dfrac{\sin x}{x}$ is the function $F(x) = 1, x \in (-1, 1)$ and $F(x) = 0$ outside $(-1, 1)$. Hint: Use the inversion theorem formally.

4. Let $R^2(\mathbb{R})$ be the vector space of all functions f on the real line whose square is improperly integrable. Show that $R^2(\mathbb{R})$ is not a subset of $R(\mathbb{R})$ and $R(\mathbb{R})$ is not a subset of $R^2(\mathbb{R})$. This is in contrast with Riemann integration in which the square of any integrable function is integrable and the square root of any integrable function is integrable. Hint: Consider the functions $f(x) = 1/\sqrt{x}$ and $g(x) = 1/x,\ x \geq 1$.